ON INVARIANTS OF A CONVECTIVE DRYING PROCESS

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The Lewis relation is proved to be valid in the absence of an analogy between heat and mass transfer. Invariance of temperature and equilibrium curves is established for some drying processes.

In practice the facts are known when the drying temperature coefficient and the forms of the drying curves do not depend on hydrodynamic conditions. However since there is no theoretical justification and the domains of applicability are unknown, the calculation methods using this effect may not be considered sufficiently reliable. Therefore the problem of justification of the possible existence of drying process invariants seems to be rather interesting [1-3].

Because of the diversity of the processes of external and internal heat and mass transfer the problem under consideration requires different approaches. The present article is concerned with analysis of interphase interaction mainly from the viewpoint of external heat and mass transfer that, by the way, does not exclude extending some obtained dependences to more complicated cases.

1. **Material heating in the period of a falling rate of drying.** Represent a drying process as a series of successive elementary events of the interaction between some air mass L and a moist material with dry mass G accomplished without transfer with the surrounding medium and at a constant pressure. Then in the adiabatic "gas-material" system at constant pressure the enthalpy within the limits of each elementary act does not change at moderate drying conditions and considering the additivity of the latter we have

$$dI + dI_m = 0, \quad \text{or} \quad dI = -dI_m. \tag{1}$$

A change in the enthalpy of a solid phase (per 1 kg of dry air) may be expressed as follows

$$dI_m = c_\ell \Theta_s \frac{G}{L} du - q_b \frac{G}{L} du + c_m \frac{G}{L} d\Theta. \tag{2}$$

Here the first term in the r.h.s. takes into account the physical heat of the liquid with a temperature θ_s removed from the material at the expense of evaporation (du < 0), the second term describes the heat supplied to the solid phase to overcome the energy of moisture binding with the material, the third term is the heat used for material heating. Combined solution of (1), (2) and the equation of material balance

$$\frac{G}{L} du = -dX \tag{3}$$

permits us to express to gas enthalpy in terms of solid phase parameters as

$$dI = c_\ell \Theta_s dX - q_b dX + c_m \frac{d\Theta}{du} dX. \tag{4}$$

Consider the interaction between a gas and a material having invariable parameters along the entire surface of contact that implies the constancy of all terms in the r.h.s. of (4). However, it does not mean their constancy in time, therefore the last term in the equation remains. It is obvious that a gas under the above conditions may vary from the initial state (I, X) to the state of equilibrium with a material surface (I, X). Then integrating (4) within the indicated limits, we obtain

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$$\frac{I_s - I}{X_s - X} = c_\ell \Theta_s - q_b + c_m \frac{d\Theta}{du}. \tag{5}$$

Analogous problems are solved in [4]. Note that the expression in the l.h.s. of (5) characterizes the slope of the intercept connecting points with the parameters of the gas (I, X) and the material surface (I_s, X_s) being in contact at this moment of time. Consequently, (5) is the equation of the contact line AB of phases (Fig. 1). Also, consider the tangent at an arbitrary point A to curve 1 displaying a change in a gas condition. Its slope is easy to determine from Eq. (4):

$$\frac{dI}{dX} = c_\ell \Theta_s - q_b + c_m \frac{d\Theta}{du}. \tag{6}$$

Comparison of (6) with (5) allows us to conclude on their identity. Therefore, any tangent to curve 1 is at the same time the contact line and instead of Eq. (6) the identical Eq. (5) may be used.

The latter relates the parameters of contacting phases with a temperature coefficient of drying $d\theta/du$, therefore, it may be used as the basic equation of heating. It establishes the relationship between the temperature and humidity of the material to be dried without considering real time. It is more convenient to write Eq. (5) in another form. The enthalpy difference is

$$I_s - I = (c_{d.g} + c_v X_s)\Theta_s + r_0 X_s - (c_{d.g} + c_v X)t - r_0 X. \tag{7}$$

Introduce the heat of a wet gas

$$c_g = c_{d.g} + c_v X$$

and the heat of bound moisture evaporation at the temperature θ_s

$$r = r_0 - c_\ell \Theta_s + c_v \Theta_s + q_b. \tag{8}$$

Then Eq. (5) acquires the form

$$c_m \frac{d\Theta}{du} = r - c_g \frac{t - \Theta_s}{X_s - X}; \quad \mathrm{Rb} = \frac{c_g}{r} \frac{t - \Theta_s}{X_s - X} - 1. \tag{9}$$

It is worth noting that by the parameters t and X are understood the quantities averaged in a gas flow cross section with due regard for the volume of the boundary layer.

The heating equation may be expressed in a different way. The heat balance for 1 kg of dried mass may be written in the form

$$\alpha f (t - \Theta_m) = c_m \frac{d\Theta}{du} \frac{du}{d\tau} - r \frac{du}{d\tau}. \tag{10}$$

Express the rate of drying as

$$-\frac{du}{d\tau} = \beta f (X_s - X). \tag{11}$$

Combined solution of (11) and (10) yields

$$c_m \frac{d\Theta}{du} = r - \frac{\alpha}{\beta} \frac{t - \Theta_s}{X_s - X}. \tag{12}$$

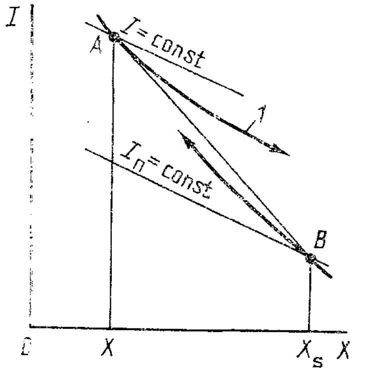

Fig. 1

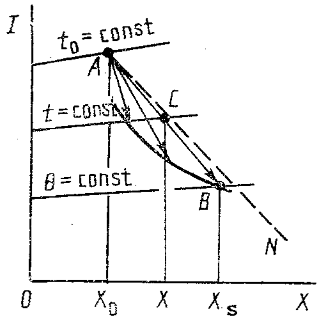

Fig. 2

Fig. 1. Processes of the moist gas–material interaction in the I–X diagram.

Fig. 2. Variation of the condition of contacting phases in a layer under mixing.

Having compared (12) with (9), we may write

$$\alpha/\beta = c_g. \tag{13}$$

The relation (13) is the modified form of the known Lewis relation [6]. It testifies to a rigorous relationship between heat and mass fluxes under drying conditions. True, the opinion is widely spread that the Lewis relation is limited by the region of existence of the analogy between the mentioned fluxes. However on deriving (13), no conditions have been imposed to result in the necessity of analogy, therefore the above constraint needs checking. Consider a heat flux passing across 1 m^2 of the contact surface of phases. In the case of smooth variation of the process parameters the flux is described by the equation

$$-rD\frac{\partial X'}{\partial y|_{y=0}} + \frac{c_m}{f}\frac{d\Theta}{du}\frac{du}{d\tau} = \lambda\frac{\partial t'}{\partial y|_{y=0}}. \tag{14}$$

The first term here describes the heat consumed for moisture evaporation, the second term — for material heating, the r.h.s represents the supplied heat flux. A rate of drying may be expressed in terms of the mass flow at the boundary as

$$-\frac{du}{d\tau} = fD\frac{\partial X'}{\partial y|_{y=0}}. \tag{15}$$

Finally, we obtain

$$-D\frac{\partial X'}{\partial y|_{y=0}}\left(r - c_m\frac{d\Theta}{du}\right) = \lambda\frac{\partial t'}{\partial y|_{y=0}}. \tag{16}$$

Introduce dimensionless temperature and moisture content

$$t^* = \frac{t - t'}{t - \Theta_s}; \quad X^* = \frac{X' - X}{X_s - X}.$$

Then

$$D(X_s - X)\frac{\partial X^*}{\partial y|_{x=0}}\left(r - c_m\frac{d\Theta}{du}\right) = \lambda(t - \Theta_s)\frac{\partial t^*}{\partial y|_{y=0}}. \tag{17}$$

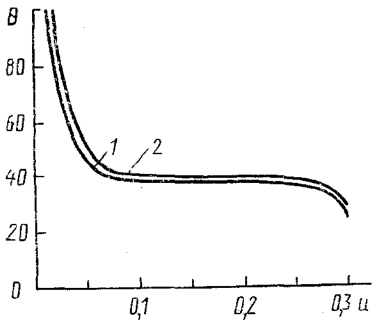

Fig. 3

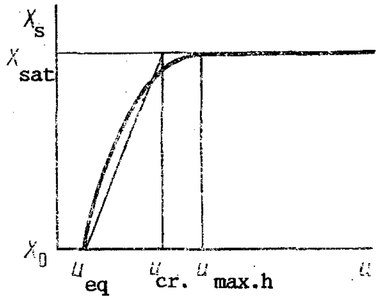

Fig. 4

Fig. 3. Temperature curves of nongradient drying for wood at $t_0 = 100°C$; 1) $X_0 = 0.006$ kg/kg; 2) 0.014. θ, °C; u, kg/kg.

Fig. 4. Typical curve of equilibrium air moisture content on a surface of a material under drying.

If (9) and, consequently, (13) are valid, then substituting (9) into (17) and canceling terms, we arrive at

$$a_m \frac{\partial X^*}{\partial y|_{y=0}} = a \frac{\partial t^*}{\partial y|_{y=0}}, \tag{18}$$

where $a_m = D/\rho_g$, $a = \lambda(c_g\rho_g)$.

Thus, the slopes of the dimensionless profiles of the temperature and the gas moisture content near the surface are inversely proportional to the corresponding transfer coefficients. Coincidence of the profiles (analogy) may take place at $a_m = a$ that is not necessary at all to fulfill the equality (18). In the case of nonuniform gas flow around solid particles, consideration must be given to the particle surface-averaged gradients. Then the fulfillment of (18) at local points of contact of the phases is not assured. This relation may be written in the following identical form

$$\frac{\alpha l}{a\rho_g c_g} = \frac{\beta l D}{D a \rho_g} \quad \text{or} \quad \mathrm{Nu}_m = \mathrm{Nu}\frac{a}{a_m}. \tag{19}$$

The expression (19) allows calculation of a mass transfer Nusselt number by the known heat Nusselt number and vice versa. Whence it also follows that the analogy is not necessary. Thus Eq. (13) may serve as one of the invariants of the drying process.

Below only those cases will be considered when we may neglect temperature and moisture content gradients inside material particles under drying. For the sake of brevity, such processes will be referred to as the nongradient processes. Then the mean temperature and moisture content of particles coincide with their values on the surface, i.e. $u_s \approx u$, $\theta_s \approx \theta$.

Air moisture content on the surface of particles is unambiguously determined by the surface moisture content and its temperature using the equation for surface equilibrium

$$X_s = X(u, \Theta). \tag{20}$$

When single particles or a thin (elementary) material layer are to be dried, the air parameters are, in fact, constant and equal to the original values: $X = X_0$, $t = t_0$, then the heating equation

$$c_m \frac{d\Theta}{du} = r - c_g \frac{t_0 - \Theta}{X(u, \Theta) - X_0} \tag{21}$$

at the initial condition $\theta = \theta_0$, $u = u_0$ has a single solution. Therefore, the temperature dependence $\theta = \theta(u)$ obtained from solution of (21) is the unified relation for all the hydrodynamic conditions.

Next, consider the interaction between a gas and a sufficiently high material layer being mixed with respect to its thickness. Owing to the mixing, the temperature and moisture content of the material may be considered to be equal with respect to height. Then change of the gas condition in the I−X diagram according to (5) will be represented by an intercept connecting the points with the coordinates (X_0, t_0) and (X_s, θ) (Fig. 2). A slope changes with time as far as the parameters X_s and θ change.

From Eq. (9) it follows that, with other conditions being equal, $d\theta/du$ depends on $(t - \theta)/(X_s - X)$. Considering the latter with reference to Fig. 2, note that it characterizes the slope of the segment CB where position of the point C depends on the path traversed by the gas in the layer. At the layer inlet, the point C coincides with the initial point A. As far as it penetrates deep into the layer, it approaches B characterizing air parameters on the surface of particles. But in any case, the slopes of the segments CB and AB are equal, therefore the following relations are also equal

$$\frac{t-\Theta}{X_s - X} = \frac{t_0 - \Theta}{X_s - X_0}. \qquad (22)$$

Thus, for a mixed layer of any height, Eq. (21) for a thin layer remains valid, which proves the independence of the temperature curve form on the material's mass and the gas flow rate. This property allows us to consider the temperature dependence $\theta = \theta(u)$ under nongradient drying conditions as an important integrated characteristic of the product, i.e., some kind of an invariant. It is worth noting that invariance is valid only with respect to hydrodynamic, but not inlet, factors: X_0, t_0, u_0, θ_0 and is not confirmed at conditions differing from those in the case of a mixed or thin layer [5]. A typical form of the temperature curves at nongradient drying conditions is shown in Fig. 3.

From Eq. (21) it follows that in dependence on the combination of X_s and θ either material heating or cooling may proceed. Illustrate it with the aid of the I−X diagram. Draw the straight line AN with the slope $\Delta t/\Delta X = r$ from the point A (Fig. 2) with the coordinates (X_0, t_0). This line subdivides the entire space of the diagram into two regions. In the region to the left of it $(t_0 - \theta)/(X_s - X_0) > r$ and $d\theta/du < 0$ take place, i.e., material heating proceeds. On the contrary, any particle, the parameters of which X_s and θ correspond to the region to the right from AN, gets cooled despite the fact that the gas temperature is higher as compared to the material particles. Points on the very line AN correspond to the processes with a constant particle temperature. As far as the material moisture content decreases, the total heat of moisture evaporation increases, therefore the slope of AN will increase in some time.

2. **On the equations of drying kinetics.** The existence, in the discussed conditions, of the temperature function $\theta = \theta(u)$, independent on hydrodynamics of the process leads, according to (20), to the unambiguous dependence of the air moisture content over the contact surface of phases X_s on humidity. This dependence is also the invariant of the drying process alone with the curve of partial vapor pressures. The typical plot of X_s on u is shown in Fig. 4. Invariance of this curve allows a judgement about it in a certain sense as an equilibrium curve.

At a moisture content larger than $u_{max.h}$, $X_s = X_{sat}$, the drying rate has its maximum according to (11), and constant value:

$$-\left(\frac{du}{d\tau}\right)_1 = N = \beta_1 f\,(X_{sat} - X_1). \qquad (23)$$

At a humidity smaller than $u_{max.h}$, the moisture content on the surface X_s decreases, which in accordance with (11) results in a drastic decrease of a drying rate. Thus, a falling rate of drying in the 2nd period is not obligatorily related with the deepening of an evaporation zone or in engaging the mechanism of internal moisture diffusion but it is also specified by thermodynamics of the particle surface.

Write the drying rate equation (11) in the following form

$$-\frac{du}{d\tau} = \beta_1 f\,(X_{sat} - X_1)\,\frac{\beta}{\beta_1}\,\frac{X_s - X}{X_{sat} - X_1}. \qquad (24)$$

Here β_1 and X_1 are referred to the period of the constant drying rate. In the case of a thin layer

$$X_s - X \approx X_s - X_0; \quad X_{sat} - X_1 \approx X_{sat} - X_0. \tag{25}$$

Replace the curvilinear section of the equilibrium curve by a straight line with the coordinates of the utmost points X_0, u_{eq} and X_{sat}, $u_{cr.red.}$ (Fig. 4). Then we may use the relation

$$\frac{X_s - X_0}{X_{sat} - X_0} \approx \frac{u - u_{eq}}{u_{c.r.} - u_{eq}}. \tag{26}$$

Designating

$$\chi = \frac{\beta}{\beta_1} \frac{1}{u_{c.r.} - u_{eq}}, \tag{27}$$

with account of (23), we arrive at the known drying relation proposed by A. V. Lykov [1]:

$$-\frac{du}{d\tau} = N\chi (u - u_p). \tag{28}$$

In real drying processes, the mass transfer coefficient β continuously decreases because of unsteadiness of the process and also of a different degree of the disturbing effect of a vapor flow on a boundary layer. Therefore, strictly speaking, the relative drying coefficient χ is not constant. In such cases, the methods of piecewise-linear approximation are used or a value averaged in the entire range is taken. In practice, χ is usually determined not from the equilibrium curve but from an experimental drying curve that is absolutely identical.

Representation of the drying equation in the form of the linearly decreasing dependence (28) has certain merits. At constant u_{eq}, it is easily integrated

$$N\tau = \frac{1}{\chi} \ln \frac{u_{c.r.} - u_{eq}}{u_f - u_{eq}} \tag{29}$$

Then with the same final humidity u_f, we have

$$N_1\tau_1 = N_2\tau_2 = N\tau = \text{const}. \tag{30}$$

This property underlies the method of correlation of experimental data, according to which drying curves obtained at different conditions may be correlated as one curve built in the coordinates $u - N\tau$ [7].

Another method may be obtained by writing (29) for different final humidities and taking their ratio:

$$\frac{\tau_1}{\tau_2} = \ln \frac{u_{c.r.} - u_{eq}}{u_1 - u_{eq}} \Big/ \ln \frac{u_{c.r.} - u_{eq}}{u_2 - u_{eq}}. \tag{31}$$

The r.h.s. in (31) does not depend on drying conditions, therefore at equal u_1 and u_2 we may write for different drying conditions

$$\frac{\tau_1}{\tau_2} = \frac{\tau_1'}{\tau_2'} = \frac{\tau_1''}{\tau_2''} = \text{const}. \tag{32}$$

An equation for drying kinetics may be obtained also by approximating the temperature curve. Replacing, e.g., the curvilinear section of the invariant temperature dependence by an intercept, we may again arrive at Eq. (28), in this case $d\theta/du$ = const and Rb = const.

Along with the discussed methods, use has been made of the methods of approximation of invariant curves by any nonlinear relation [7]. A drawback of all the approximation methods used to describe the drying kinetics is their limitedness by the domains of invariance and condition (25). In this sense, the kinetics equation in the general form (11) is devoid of the above drawbacks.

NOTATION

I, enthalpy; c, heat capacity; t, θ, temperature of the gas and a material, respectively; u, u_s, $u_{cr.red}$, $u_{max.h}$, u_{eq}, mean, surface, critical reduced, maximum hygroscopic, equilibrium moisture content of the material, respectively; X, air moisture content; q_b, binding energy of moisture with the material; r_0, heat of liquid evaporation at 0°C; D, vapor diffusion coefficient in the gas calculated from the moisture content gradient; f, specific contact surface of phases, m^2/kg; l, determining dimension; X', t', moisture content and temperature of the gas in the boundary layer; y, normal to the contact surface of phases; α, β, interphase heat and mass transfer coefficients; λ, gas thermal conductivity; ρ, density; τ, time; $Rb = c_m(-d\theta/du)r$, Reminder number; $Nu = \alpha l/\lambda$, $Nu_m = \beta l/D$, heat and mass transfer Nusselt numbers. Indices: g, gas, m, material; s, surface; d.g, dry gas; l, liquid; v, vapor; sat, saturated, w.t, wet-bulb thermometer; f, final, index 0 stands for parameters at the dryer inlet.

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